MATH HANDBOOK

Mathematics is a language used in science to express and solve problems. Calculations you perform during your study of chemistry require arithmetic operations, such as addition, subtraction, multiplication, and division. Use this handbook to review basic math skills and to reinforce some math skills presented in more depth in the chapters.

Scientific Notation

Scientists must use extremely small and extremely large numbers to describe the objects in **Figure 1.** The mass of the proton at the center of a hydrogen atom is 0.00000000000000000000000001673 kg. HIV, the virus that causes AIDS, is about 0.00000011 m. The temperature at the center of the Sun reaches 15,000,000 K. Such small and large numbers are difficult to read and hard to work with in calculations. Scientists have adopted a method of writing exponential numbers called scientific notation. It is easier than writing numerous zeros when numbers are very large or very small. It is also easier to compare the relative size of numbers when they are written in scientific notation.

A number written in scientific notation has two parts.
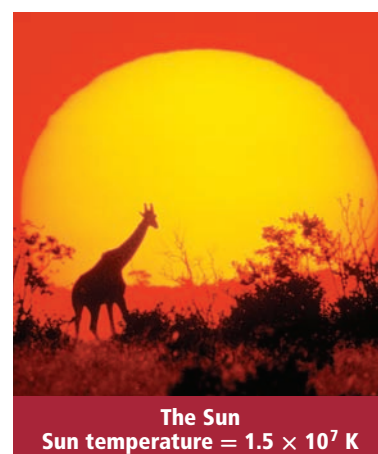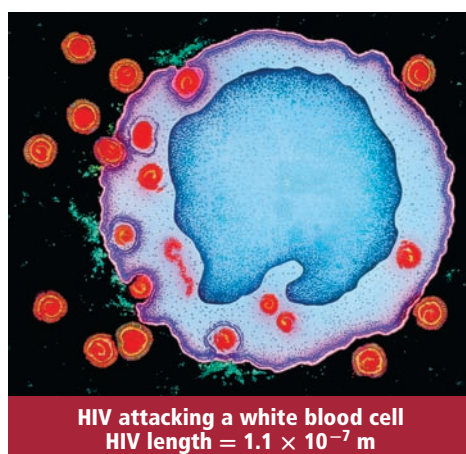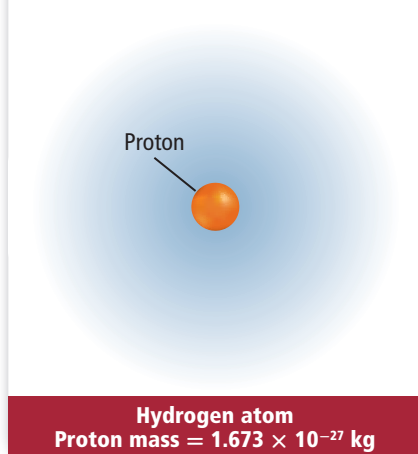
$$N \times 10^n$$

The first part (N) is a number in which only one digit is placed to the left of the decimal point and all remaining digits are placed to the right of the decimal point. The second part is an exponent of ten (10^n) by which the decimal portion is multiplied. For example, the number 2.53×10^6 is written in scientific notation.

$$2.53 \times 10^6$$

Number between one and ten   Exponent of ten

The decimal portion is 2.53 and the exponent is 10^6.

Positive exponents are used to express large numbers, and negative exponents are used to express small numbers.

■ **Figure 1** Scientific notation provides a convenient way to express data with extremely large or small numbers. Scientists can express the mass of a proton, the length of HIV, and the temperature of the Sun in scientific notation.

Hydrogen atom
Proton mass = 1.673×10^{-27} kg

HIV attacking a white blood cell
HIV length = 1.1×10^{-7} m

The Sun
Sun temperature = 1.5×10^7 K

Positive exponents

When scientists discuss the physical properties of the Moon, shown in **Figure 2,** the numbers are enormously large. A positive exponent of 10, (n) tells how many times a number must be multiplied by 10 to give the long form of the number.

$$2.53 \times 10^6$$
$$= 2.53 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$$
$$= 2,530,000$$

You can also think of the positive exponent of 10 as the number of places you move the decimal to the left until only one nonzero digit is to the left of the decimal point.

2,530,000.　　　　**The decimal point moves six places to the left.**

To convert the number 567.98 to scientific notation, first write the number as an exponential number by multiplying by 10^0.

$$567.98 \times 10^0$$

(Remember that multiplying any number by 10^0 is the same as multiplying the number by 1.) Move the decimal point to the left until there is only one digit to the left of the decimal. At the same time, increase the exponent by the same number as the number of places the decimal is moved.

$567.98 \times 10^{0+2}$　　　　**The decimal point moves two places to the left.**

Thus, 567.98 written in scientific notation is 5.6798×10^2.

Negative exponents

Measurements can also have negative exponents, such as shown by the X-rays in **Figure 3.** Negative exponents are used for numbers that are very small. A negative exponent of 10 tells how many times a number must be divided by 10 to give the long form of the number.

$$6.43 \times 10^{-4} = \frac{6.43}{10 \times 10 \times 10 \times 10} = 0.000643$$

A negative exponent of 10 is the number of places you move the decimal to the right until it is just past the first nonzero digit.

When converting a number that requires the decimal to be moved to the right, the exponent is decreased by the appropriate number. For example, the expression of 0.0098 in scientific notation is as follows:

0.0098×10^0

$0.0098 \times 10^{0-3}$　　　　**The decimal point moves three places to the right.**

9.8×10^{-3}

Thus, 0.0098 written in scientific notation is 9.8×10^{-3}.

■ **Figure 2** The mass of the Moon is 7.349×10^{22} kg.

■ **Figure 3** Because of their short wavelengths (10^{-8} m to 10^{-13} m), X-rays can pass through some objects.

(t)Julian Baum/Science Photo Library/Photo Researchers, (b)David Frazier/CORBIS

Operations with Scientific Notation

The arithmetic operations performed with ordinary numbers can be done with numbers written in scientific notation. However, the exponential portion of the numbers must also be considered.

1. Addition and subtraction

Before numbers in scientific notation can be added or subtracted, the exponents must be equal. Remember that the decimal is moved to the left to increase the exponent and to the right to decrease the exponent.

$$(3.4 \times 10^2) + (4.57 \times 10^3) = (0.34 \times 10^3) + (4.57 \times 10^3)$$
$$= (0.34 + 4.57) \times 10^3$$
$$= 4.91 \times 10^3$$

$$(7.52 \times 10^{-4}) - (9.7 \times 10^{-5}) = (7.52 \times 10^{-4}) - (0.97 \times 10^{-4})$$
$$= (7.52 - 0.97) \times 10^{-4}$$
$$= 6.55 \times 10^{-4}$$

2. Multiplication

When numbers in scientific notation are multiplied, only the decimal portion is multiplied. The exponents are added.

$$(2.00 \times 10^3)(4.00 \times 10^4) = (2.00)(4.00) \times 10^{3+4}$$
$$= 8.00 \times 10^7$$

3. Division

When numbers in scientific notation are divided, only the decimal portion is divided, while the exponents are subtracted as follows:

$$\frac{9.60 \times 10^7}{1.60 \times 10^4} = \frac{9.60}{1.60} \times 10^{7-4}$$
$$= 6.00 \times 10^3$$

PRACTICE Problems
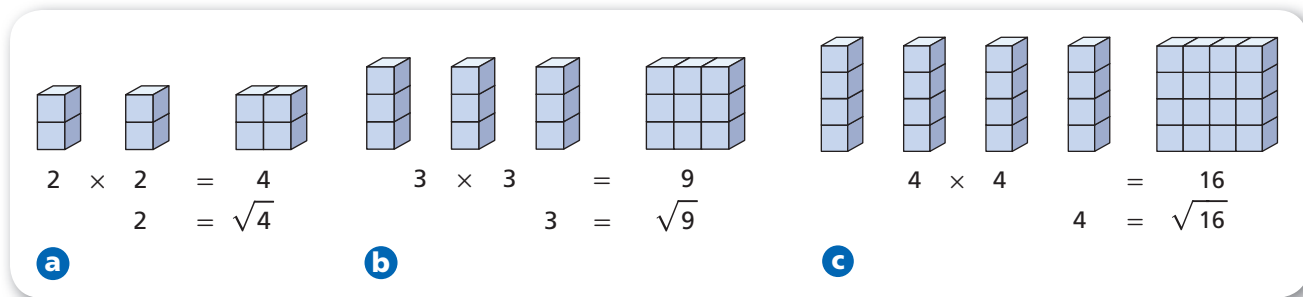
1. Express the following numbers in scientific notation.
   a. 5800
   b. 453,000
   c. 0.0005877
   d. 0.0036

2. Perform the following operations.
   a. $(5.0 \times 10^6) + (3.0 \times 10^7)$
   b. $(1.8 \times 10^9) + (2.0 \times 10^8)$
   c. $(3.89 \times 10^{12}) - (1.9 \times 10^{11})$
   d. $(6.0 \times 10^{-8}) - (4.0 \times 10^{-9})$

3. Perform the following operations.
   a. $(6.0 \times 10^{-4}) \times (4.0 \times 10^{-6})$
   b. $(4.5 \times 10^9) \times (6.0 \times 10^{-10})$
   c. $\dfrac{4.5 \times 10^{-8}}{1.5 \times 10^{-4}}$
   d. $\dfrac{9.6 \times 10^8}{1.6 \times 10^{-6}}$
   e. $\dfrac{(2.5 \times 10^6)(7.2 \times 10^4)}{1.8 \times 10^{-5}}$
   f. $\dfrac{(6.2 \times 10^{12})(6.0 \times 10^{-7})}{1.2 \times 10^6}$

$2 \times 2 = 4$
$2 = \sqrt{4}$

a

$3 \times 3 = 9$
$3 = \sqrt{9}$

b

$4 \times 4 = 16$
$4 = \sqrt{16}$

c

Square and Cube Roots

A square root is one of two identical factors of a number. As shown in **Figure 4a,** the number 4 is the product of two identical factors—2. Thus, the square root of 4 is 2. The symbol $\sqrt{}$, called a radical sign, is used to indicate a square root. Most scientific calculators have a square root key labeled $\sqrt{}$.

$$\sqrt{4} = \sqrt{2 \times 2} = 2$$

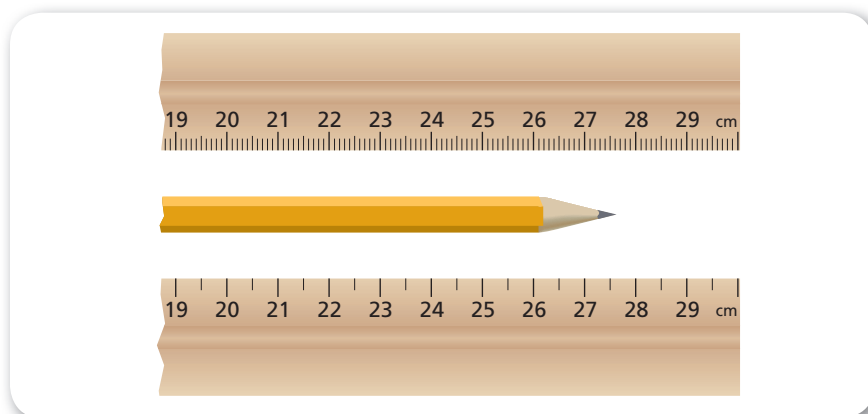This equation is read "the square root of 4 equals 2." What is the square root of 9, shown in **Figure 4b?**

There can be more than two identical factors of a number. You know that $2 \times 4 = 8$. Are there any other factors of the number 8? It is the product of $2 \times 2 \times 2$. A cube root is one of three identical factors of a number. Thus, what is the cube root of 8? It is 2. A cube root is also indicated by a radical.

$$\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$$

Check your calculator handbook for more information on finding roots.

■ **Figure 4 a.** The number 4 can be expressed as two groups of 2. The identical factors are 2. **b.** The number 9 can be expressed as three groups of 3. Thus, 3 is the square root of 9. **c.** 4 is the square root of 16.

Determine *the cube root of 16 using your calculator.*

Significant Figures

Accuracy reflects how close the measurements you make in the laboratory come to the real value. Precision describes the degree of exactness of your measurements. Which ruler in **Figure 5** would give you the most precise length? The top ruler, with the millimeter markings, would allow your measurements to come closer to the actual length of the pencil. The measurement would be more precise.
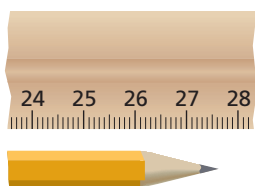

■ **Figure 5** The estimated digit must be read between the millimeter markings on the top ruler.

Evaluate *Why is the bottom ruler less precise?*

■ **Figure 6** If you determine that the length of this pencil is 27.65 cm, that measurement has four significant figures.

Measuring tools are never perfect, nor are the people doing the measuring. Therefore, whenever you measure a physical quantity, there will always be some amount of uncertainty in the measurement. The number of significant figures in the measurement indicates the uncertainty of the measuring tool.

The number of significant figures in a measured quantity is all of the certain digits plus the first uncertain digit. For example, the pencil in **Figure 6** has a length that is between 27.6 and 27.7 cm. You can read the ruler to the nearest millimeter (27.6 cm), but after that you must estimate the next digit in the measurement. If you estimate that the next digit is 5, you would report the measured length of the pencil as 27.65 cm. Your measurement has four significant figures. The first three are certain, and the last is uncertain. The ruler used to measure the pencil has precision to the nearest tenth of a millimeter.

How many significant figures?

When a measurement is provided, the following series of rules will help you to determine how many significant figures there are in that measurement.

1. *All nonzero figures are significant.*

2. *When a zero falls after the decimal point and after a significant figure, that zero is significant.*

3. *When a zero falls between significant figures, the zero is also significant.*

4. *When a zero is used merely to indicate the position of the decimal, it is not significant.*

5. *All counting numbers and exact numbers are treated as if they have an infinite number of significant figures.*

Examine each of the following measurements. Use the rules above to check that all of them have three significant figures.

245 K	**Rule 1**
18.0 L	**Rule 2**
308 km	**Rule 3**
0.00623 g	**Rule 4**
186,000 m	**Rule 4**

Suppose you must do a calculation using the measurement 200 L. You cannot be certain which zero was estimated. To indicate the significance of digits, especially zeros, write measurements in scientific notation. In scientific notation, all digits in the decimal portion are significant. Which measurement is most precise?

$$200 \text{ L has one significant figure.}$$
$$2 \times 10^2 \text{ L has one significant figure.}$$
$$2.0 \times 10^2 \text{ L has two significant figures.}$$
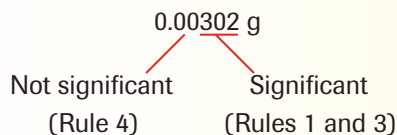$$2.00 \times 10^2 \text{ L has three significant figures.}$$

The greater the number of digits in a measurement expressed in scientific notation, the more precise the measurement is. In this example, 2.00×10^2 L is the most precise data.

EXAMPLE Problem 1

SIGNIFICANT FIGURES How many significant figures are in the measurement 0.00302 g? 60 min? 5.620 m? 9.80×10^2 m/s^2?

1 ANALYZE THE PROBLEM

To determine the number of significant digits in a series of numbers, review the rules for significant figures.

2 SOLVE FOR THE UNKNOWN

0.00302 g

Not significant   Significant
(Rule 4)   (Rules 1 and 3)

The measurement 0.00302 g has three significant figures.

60 min
Unlimited significant figures
(Rule 5)

5.620 m
Significant
(Rules 1 and 2)

The measurement 5.620 m has four significant figures.

9.80×10^2 m/s^2
Significant
(Rules 1 and 2)

3 EVALUATE THE ANSWER

The measurements 0.00302 g and 9.80×10^2 m/s^2 have three significant figures. The measurement 60 min has unlimited significant figures. The measurement 5.620 m has four significant figures.

PRACTICE Problems

4. Determine the number of significant figures in each measurement:

   a. 35 g
   b. 3.57 m
   c. 3.507 km
   d. 0.035 kg
   e. 0.246 L
   f. 0.004 m^3
   g. 24.068 kPa
   h. 268 K
   i. 20.04080 g
   j. 20 dozen
   k. 730,000 kg
   l. 6.751 g
   m. 0.157 kg
   n. 28.0 mL
   o. 2500 m
   p. 0.070 mol
   q. 30.07 nm
   r. 0.106 cm
   s. 0.0076 g
   t. 0.0230 cm^3
   u. 26.509 cm
   v. 54.52 cm^3
   w. 2.40×10^6 kg
   x. 4.07×10^{16} m

■ **Figure 7** Compare the markings on the graduated cylinder at the top with the markings on the beaker at the bottom.

Analyze *Which piece of glassware will yield more precise measurements?*

Rounding

Arithmetic operations that involve measurements are done the same way as operations involving any other numbers. However, the results must correctly indicate the uncertainty in the calculated quantities. Perform all of the calculations, and then round the result to the least number of significant figures in any of the measurements used in the calculations. To round a number, use the following rules.

1. *When the left most digit to be dropped is less than 5, that digit and any digits that follow are dropped. Then, the last digit in the rounded number remains unchanged.* For example, when rounding the number 8.7645 to three significant figures, the left most digit to be dropped is 4. Therefore, the rounded number is 8.76.

2. *When the left most digit to be dropped is greater than 5, that digit and any digits that follow are dropped, and the last digit in the rounded number is increased by one.* For example, when rounding the number 8.7676 to three significant figures, the left most digit to be dropped is 7. Therefore, the rounded number is 8.77.

3. *When the left most digit to be dropped is 5 followed by a nonzero number, that digit and any digits that follow are dropped. The last digit in the rounded number increases by one.* For example, 8.7519 rounded to two significant figures equals 8.8.

4. *If the digit to the right of the last significant figure is equal to 5 and is not followed by a nonzero digit, look at the last significant figure. If it is odd, increase it by one; if even, do not round up.* For example, 92.350 rounded to three significant figures equals 92.4, and 92.25 equals 92.2.

Calculations with significant figures

Look at the glassware in **Figure 7.** Would you expect to measure a more precise volume with the beaker or the graduated cylinder? When you perform any calculation using measured quantities such as volume or mass, it is important to remember that the result can never be more precise than the least-precise measurement. That is, your answer cannot have more significant figures than the least precise measurement. Note that it is important to perform all calculations before dropping any insignificant digits.

The following rules determine how to use significant figures in calculations that involve measurements.

1. *To add or subtract measurements, first perform the mathematical operation, then round off the result to the least-precise value.* There should be the same number of digits to the right of the decimal as the measurement with the least number of decimal digits.

2. *To multiply or divide measurements, first perform the calculation, then round the answer to the same number of significant figures as the measurement with the least number of significant figures.* The answer should contain no more significant figures than the fewest number of significant figures in any of the measurements in the calculation.

EXAMPLE Problem 2

CALCULATING WITH SIGNIFICANT FIGURES Air contains oxygen (O_2), nitrogen (N_2), carbon dioxide (CO_2), and trace amounts of other gases. Use the known pressures in **Table 1** to calculate the partial pressure of oxygen.

1 ANALYZE THE PROBLEM

The data in **Table 1** contains the gas pressure for nitrogen gas, carbon dioxide gas, and trace gases. To add or subtract measurements, first perform the operation, then round off the result to correspond to the least-precise value involved.

2 SOLVE FOR THE UNKNOWN

$P_{O_2} = P_{total} - \left(P_{N_2} + P_{CO_2} + P_{trace}\right)$

$P_{O_2} = 101.3 \text{ kPa} - \left(79.10 \text{ kPa} + 0.040 \text{ kPa} + 0.94 \text{ kPa}\right)$

$P_{O_2} = 101.3 \text{ kPa} - 80.080 \text{ kPa}$

$P_{O_2} = 21.220 \text{ kPa}$

The total pressure $\left(P_{total}\right)$ was measured to the tenths place. It is the least precise measurement. Therefore, the result should be rounded to the nearest tenth of a kilopascal. The pressure of oxygen is $P_{O_2} = 21.2$ kPa.

3 EVALUATE THE ANSWER

By adding the gas pressure of all the gases, including oxygen, the total gas pressure is 101.3 kPa.

Table 1 Pressures of Gases in Air	
	Pressure (kPa)
Nitrogen gas	79.10
Carbon dioxide gas	0.040
Trace gases	0.94
Total gases	101.3

PRACTICE Problems

5. Round off the following measurements to the number of significant figures indicated in parentheses.
   a. 2.7518 g (3)
   b. 8.6439 m (2)
   c. 13.841 g (2)
   d. 186.499 m (5)
   e. 634,892.34 (4)
   f. 355,500 g (2)

6. Perform the following operations.
   a. $(2.475 \text{ m}) + (3.5 \text{ m}) + (4.65 \text{ m})$
   b. $(3.45 \text{ m}) + (3.658 \text{ m}) + (47 \text{ m})$
   c. $\left(5.36 \times 10^{-4} \text{ g}\right) - \left(6.381 \times 10^{-5} \text{ g}\right)$
   d. $\left(6.46 \times 10^{12} \text{ m}\right) - \left(6.32 \times 10^{11} \text{ m}\right)$
   e. $\left(6.6 \times 10^{12} \text{ m}\right) \times \left(5.34 \times 10^{18} \text{ m}\right)$
   f. $\dfrac{5.634 \times 10^{11} \text{ m}}{3.0 \times 10^{12} \text{ m}}$
   g. $\dfrac{\left(4.765 \times 10^{11} \text{ m}\right)\left(5.3 \times 10^{-4} \text{ m}\right)}{7.0 \times 10^{-5} \text{ m}}$

Solving Algebraic Equations

When you are given a problem to solve, it often can be written as an algebraic equation. You can use letters to represent measurements or unspecified numbers in the problem. The laws of chemistry are often written in the form of algebraic equations. For example, the ideal gas law relates pressure, volume, moles, and temperature of the gases. The ideal gas law is written as follows.

$$PV = n\text{R}T$$

The variables are pressure (P), volume (V), number of moles (n), and temperature (T). R is a constant. This is a typical algebraic equation that can be manipulated to solve for any of the individual variables.

When you solve algebraic equations, any operation that you perform on one side of the equal sign must be performed on the other side of the equation. Suppose you are asked to use the ideal gas law to find the pressure of a gas (P). To solve for, or isolate, P requires you to divide the left-hand side of the equation by V. This operation must be performed on the right-hand side of the equation as well, as shown in the second equation below.

$$PV = n\text{R}T$$

$$\frac{PV}{V} = \frac{n\text{R}T}{V}$$

The Vs on the left-hand side of the equation cancel each other out.

$$\frac{PV}{V} = \frac{n\text{R}T}{V}$$

$$P \times \frac{\cancel{V}}{\cancel{V}} = \frac{n\text{R}T}{V}$$

$$P = \frac{n\text{R}T}{V}$$

The ideal gas law equation is now written in terms of pressure. That is, P has been isolated.

Order of operations

When isolating a variable in an equation, it is important to remember that arithmetic operations have an order of operations, as shown in **Figure 8,** that must be followed. Operations in parentheses (or brackets) take precedence over multiplication and division, which in turn take precedence over addition and subtraction. For example, in the following equation
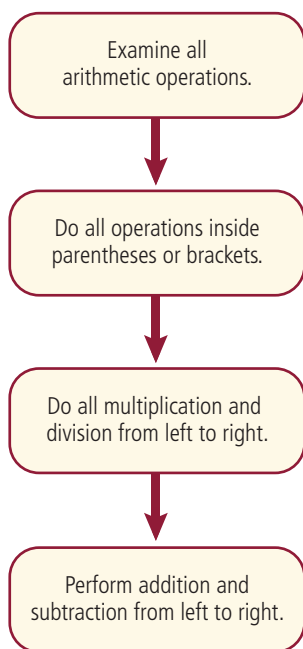
$$a + b \times c$$

variable b must be multiplied first by variable c. Then, the resulting product is added to variable a. If the equation is written

$$(a + b) \times c$$

the operation in parentheses or brackets must be done first. In the equation above, variable a is added to variable b before the sum is multiplied by variable c.

■ **Figure 8** When faced with an equation that contains more than one operation, use this flowchart to determine the order in which to perform your calculations.

Order of Operations

Examine all arithmetic operations.

Do all operations inside parentheses or brackets.

Do all multiplication and division from left to right.

Perform addition and subtraction from left to right.

To see the difference order of operations makes, try replacing a with 2, b with 3, and c with 4.

$$a + (b \times c) = 2 + (3 \times 4) = 14$$

$$(a + b) \times c = (2 + 3) \times 4 = 20$$

To solve algebraic equations, you also must remember the distributive property. To remove parentheses to solve a problem, any number outside the parentheses is distributed across the parentheses as follows.

$$6(x + 2y) = 6(x) + 6(2y) = 6x + 12y$$

EXAMPLE Problem 3

ORDER OF OPERATIONS The temperature on a cold day was 25°F. What was the temperature on the Celsius scale?

1 ANALYZE THE PROBLEM

The temperature in Celsius can be calculated by using the equation for converting from the Celsius temperature to Fahrenheit temperature. The Celsius temperature is the unknown variable. The known variable is 25°F.

2 SOLVE FOR THE UNKNOWN

Determine the equation for calculating the temperature in Celsius.

$$°F = \frac{9}{5}°C + 32$$

$$°F - 32 = \frac{9}{5}°C + 32 - 32$$ **Rearrange the equation to isolate °C. Begin by subtracting 32 from both sides.**

$$°F - 32 = \frac{9}{5}°C$$

$$5 \times (°F - 32) = 5 \times \frac{9}{5}°C$$ **Then, multiply both sides by 5.**

$$5 \times (°F - 32) = 9°C$$

$$\frac{5 \times (°F - 32)}{9} = \frac{9°C}{9}$$ **Finally, divide both sides by 9.**

$$°C = \frac{5}{9}(°F - 32)$$

$$= \frac{5}{9}(25 - 32)$$ **Substitute the known Fahrenheit temperature.**

$$= -3.9°C$$

The Celsius temperature is $-3.9°C$.

3 EVALUATE THE ANSWER

To determine if the answer is correct, place the answer, $-3.9°C$, into the original equation. If the Fahrenheit temperature is 25°, the calculation was done correctly.

PRACTICE Problems

Isolate the indicated variable in each equation.

7. $PV = nRT$ for R
8. $3 = 4(x + y)$ for y
9. $z = x(4 + 2y)$ for y
10. $\frac{2}{x} = 3 + y$ for x
11. $\frac{2x + 1}{3} = 6$ for x

Dimensional Analysis

The dimensions of a measurement refer to the type of units attached to a quantity. For example, length is a dimensional quantity that can be measured in meters, centimeters, and kilometers. Dimensional analysis is the process of solving algebraic equations for units as well as numbers. It is a way of checking to ensure that you have used the correct equation, and that you have correctly applied the rules of algebra when solving the equation. It can also help you to choose and set up the correct equation, as shown on the next page, when you learn how to do unit conversions. It is good practice to make dimensional analysis a habit by always stating the units as well as the numerical values whenever substituting values into an equation.

EXAMPLE Problem 4

DIMENSIONAL ANALYSIS The sculpture in **Figure 9** is made from aluminum. The density (D) of aluminum is 2700 kg/m³. Determine the mass (m) of a piece of aluminum of volume (V) 0.20 m³.

■ **Figure 9** Aluminum is a metal that is useful from the kitchen to the sculpture garden.

1 ANALYZE THE PROBLEM

The facts of the problem are density (2700 kg/m³), volume (0.20 m³), and the density equation, $D = m/V$.

2 SOLVE FOR THE UNKNOWN

Determine the equation for mass by rearranging the density equation.

The equation for density is

$$D = \frac{m}{V}$$

$$DV = \frac{mV}{V}$$

Multiply both sides of the equation by **V**, and isolate **m**.

$$DV = \frac{\cancel{V}}{\cancel{V}} \times m$$

$$m = DV$$

$$m = (2700 \text{ kg/m}^3)(0.20 \text{ m}^3) = 540 \text{ kg}$$

Substitute the known values for **D** and **V**.

3 EVALUATE THE ANSWER

Notice that the unit m³ cancels out, leaving mass in kg, a unit of mass.

PRACTICE Problems

Determine whether the following equations are dimensionally correct. Explain.

12. $v = s \times t$ where $v = 24$ m/s, $s = 12$ m, and $t = 2$ s.

13. $R = \frac{nT}{PV}$ where R is in L·atm/mol·K, n is in mol, T is in K, P is in atm, and V is in L.

14. $t = \frac{v}{s}$ where t is in seconds, v is in m/s, and s is in m.

15. $s = \frac{at^2}{2}$ where s is in m, a is in m/s², and t is in s.

Unit Conversion

Recall that the universal unit system used by scientists is called Le Système Internationale d'Unités, or SI. It is a metric system based on seven base units—meter, second, kilogram, kelvin, mole, ampere, and candela—from which all other units are derived. The size of a unit in the metric system is indicated by a prefix related to the difference between that unit and the base unit. For example, the base unit for length in the metric system is the meter. One-tenth of a meter is a decimeter, where the prefix *deci-* means *one-tenth*. One thousand meters is a kilometer, where the prefix *kilo-* means *one thousand*.

You can use the information in **Table 2** to express a measured quantity in different units. For example, how is 65 m expressed in centimeters? **Table 2** indicates one centimeter and one-hundredth meter are equivalent, that is, 1 cm $= 10^{-2}$ m. This information can be used to form a conversion factor. A conversion factor is a ratio equal to one that relates two units. You can make the following conversion factors from the relationship between meters and centimeters. Be sure when you set up a conversion factor that the measurement in the numerator (the top of the ratio) is equivalent to the measurement in the denominator (the bottom of the ratio).

$$1 = \frac{1 \text{ cm}}{10^{-2} \text{ m}} \text{ and } 1 = \frac{10^{-2} \text{ m}}{1 \text{ cm}}$$

Table 2 Common SI Prefixes

Prefix	Symbol	Exponential Notation	Prefix	Symbol	Exponential Notation
Peta	P	10^{15}	Deci	d	10^{-1}
Tera	T	10^{12}	Centi	c	10^{-2}
Giga	G	10^{9}	Milli	m	10^{-3}
Mega	M	10^{6}	Micro	μ	10^{-6}
Kilo	k	10^{3}	Nano	n	10^{-9}
Hecto	h	10^{2}	Pico	p	10^{-12}
Deka	da	10^{1}	Femto	f	10^{-15}

Recall that the value of a quantity does not change when it is multiplied by 1. To convert 65 m to centimeters, multiply 65 m by the conversion factor for centimeters.

$$65 \text{ m} \times \frac{1 \text{ cm}}{10^{-2} \text{ m}}$$
$$= 65 \times 10^2 \text{ cm}$$
$$= 6.5 \times 10^3 \text{ cm}$$

Note the conversion factor is set up so that the unit meters cancels and the answer is in centimeters as required. When setting up a unit conversion, use dimensional analysis to check that the units cancel to give an answer in the desired units. Always check your answer to be certain the units make sense.

You make unit conversions every day when you determine how many quarters are needed to make a dollar or how many feet are in a yard. One unit that is often used in calculations in chemistry is the mole. Equivalent relationships among moles, grams, and the number of representative particles (atoms, molecules, formula units, or ions) must be used. For example, 1 mol of a substance contains 6.02×10^{23} representative particles. Try the next Example Problem to see how this information can be used in a conversion factor to determine the number of atoms in a sample of manganese.

■ **Figure 10** The mass of one mole of manganese equals 54.94 g.

Determine *How many significant figures are in this measurement?*

EXAMPLE Problem 5

UNIT CONVERSIONS One mole of manganese (Mn), shown in **Figure 10,** has a mass of 54.94 g. How many atoms are in 2.0 mol of manganese?

1 ANALYZE THE PROBLEM

You are given the mass of 1 mol of manganese. In order to convert to the number of atoms, you must set up a conversion factor relating the number of moles and the number of atoms.

2 SOLVE FOR THE UNKNOWN

The conversion factors for moles and atoms are shown below.

$$\frac{1 \text{ mol}}{6.02 \times 10^{23} \text{ atoms}} \quad \text{and} \quad \frac{6.02 \times 10^{23} \text{ atoms}}{1 \text{ mol}}$$

Choose the conversion factor that cancels units of moles and gives an answer in number of atoms.

$$2.0 \text{ mol} \times \frac{6.02 \times 10^{23} \text{ atoms}}{1 \text{ mol}} = 12.04 \times 10^{23} \text{ atoms}$$
$$= 1.2 \times 10^{24} \text{ atoms}$$

3 EVALUATE THE ANSWER

The answer is expressed in the desired units (number of atoms). It is expressed in two significant figures because the number of moles (2.0) has two significant figures.

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16. Convert the following measurements as indicated.

a. 4 m = _____cm

b. 50.0 cm = _____m

c. 15 cm = _____mm

d. 567 mg = _____g

e. 324 mL = _____L

f. 28 L = _____mL

g. 4.6×10^3 m = _____mm

h. 8.3×10^4 g = _____kg

i. 2.7×10^2 L = _____mL

j. 7.3×10^5 mL = _____L

k. 8.4×10^{10} m = _____km

l. 3.8×10^4 m^2 = _____mm^2

m. 6.9×10^{12} cm^2 = _____m^2

n. 6.3×10^{21} mm^3 = _____cm^3

o. 9.4×10^{12} cm^3 = _____m^3

p. 5.7×10^{20} cm^3 = _____km^3

Drawing Line Graphs

Scientists, such as the one shown in **Figure 11,** as well as you and your classmates, use graphing to analyze data gathered in experiments. Graphs provide a way to visualize data in order to determine the mathematical relationship between the variables in your experiment. Line graphs are used most often.

Figure 11 also shows a line graph. Line graphs are drawn by plotting variables along two axes. Plot the independent variable on the x-axis (horizontal axis), also called the abscissa. The independent variable is the quantity controlled by the person doing the experiment. Plot the dependent variable on the y-axis (vertical axis), also called the ordinate. The dependent variable is the variable that depends on the independent variable. Label the axes with the variables being plotted and the units attached to those variables.

■ **Figure 11** Once experimental data have been collected, they must be analyzed to determine the relationships between the measured variables.

This research scientist might use graphs to analyze the data she collects on ultrapure water.
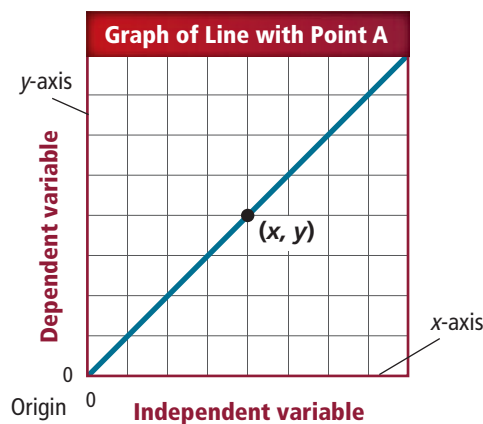
Any graph of your data should include labeled x- and y-axes, a suitable scale, and a title.

■ **Figure 12** To plot a point on a graph, place a dot at the location for each ordered pair (*x, y*) determined by your data. In the *Density of Water* graph, the dot marks the ordered pair (40 mL, 40 g). Generally, the line or curve that you draw will not include all of your experimental data points, as shown in the *Experimental Data* graph.
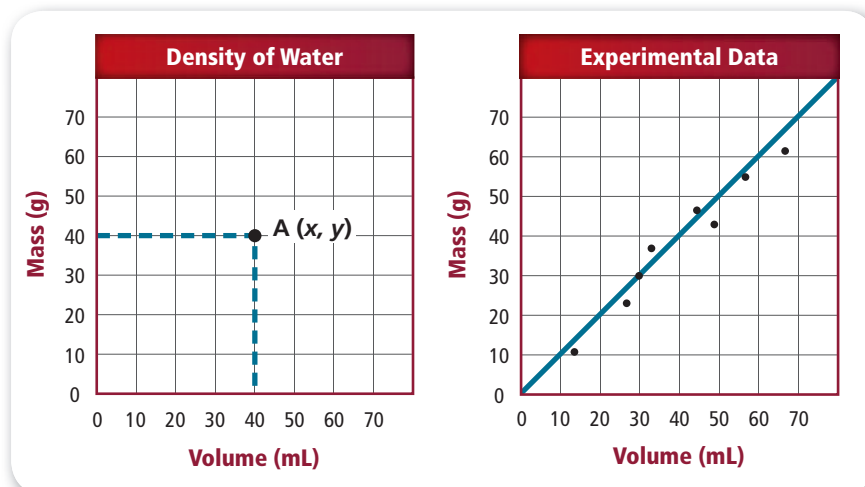
Determining a scale

An important part of graphing is the selection of a scale. Scales should be easy to plot and easy to read. First, examine the data to determine the highest and lowest values. Assign each division on the axis (the square on the graph paper) with an equal value so that all data can be plotted along the axis. Scales divided into multiples of 1, 2, 5, or 10, or decimal values, are often the most convenient. It is not necessary to start at zero, nor is it necessary to plot both variables to the same scale. Scales must, however, be labeled clearly with the appropriate numbers and units.

Plotting data

The values of the independent and dependent variables form ordered pairs of numbers, called the *x*-coordinate and the *y*-coordinate (*x, y*), that correspond to points on the graph. The first number in an ordered pair always corresponds to the *x*-axis; the second number always corresponds to the *y*-axis. The ordered pair (0,0) is always the origin. Sometimes, the points are named by using a letter. In **Figure 12,** Point A on the *Density of Water* graph corresponds to Point (*x, y*).

After the scales are chosen, plot the data. To graph or plot an ordered pair means to place a dot at the point that corresponds to the values in the ordered pair. The *x*-coordinate indicates how many units to move right (if the number is positive) or left (if the number is negative). The *y*-coordinate indicates how many units to move up or down. Which direction is positive on the *y*-axis? Negative? Locate each pair of *x*- and *y*-coordinates by placing a dot, as shown in **Figure 12** in the *Density of Water* graph. Sometimes, a pair of rulers, one extending from the *x*-axis and the other from the *y*-axis, can ensure that data are plotted correctly.

Drawing a curve

Once the data is plotted, a straight line or a curve is drawn. It is not necessary to make it go through every point plotted, or even any of the points, as shown in the *Experimental Data* graph in **Figure 12.** Graphing data is an averaging process. If the points do not fall along a line, the best-fit line or most-probable smooth curve through the points is drawn. Note that curves do not always go through the origin (0, 0).

Naming a graph

Last but not least, give each graph a title that describes what is being graphed. The title should be placed at the top of the page, or in a box on a clear area of the graph. It should not cross the data curve.
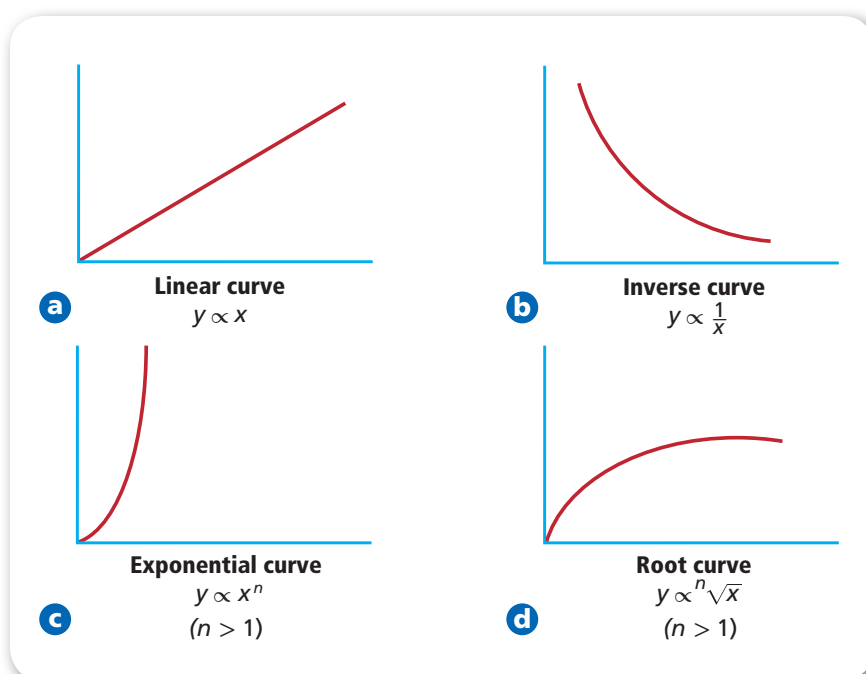
Using Line Graphs

Once the data from an experiment has been collected and plotted, the graph must be interpreted. Much can be learned about the relationship between the independent and dependent variables by examining the shape and slope of the curve. Four common types of curves are shown in **Figure 13.** Each type of curve corresponds to a mathematical relationship between the independent and dependent variables.

Direct and inverse relationships

In your study of chemistry, the most common curves are the linear, representing the direct relationship ($y \propto x$), and the inverse, representing the inverse relationship ($y \propto 1/x$), where x represents the independent variable and y represents the dependent variable. In a direct relationship, y increases in value as x increases in value, or y decreases when x decreases. In an inverse relationship, y decreases in value as x increases.

An example of a typical direct relationship is the increase in volume of a gas with increasing temperature. When the gases inside a hot-air balloon are heated, the balloon gets larger. As the balloon cools, its size decreases. However, a plot of the decrease in pressure as the volume of a gas increases yields a typical inverse curve.

You might also encounter exponential and root curves in your study of chemistry. See **Figure 13.** An exponential curve describes a relationship in which one variable is expressed by an exponent. A root curve describes a relationship in which one variable is expressed by a root.

a Linear curve
$y \propto x$

b Inverse curve
$y \propto \frac{1}{x}$

c Exponential curve
$y \propto x^n$
$(n > 1)$

d Root curve
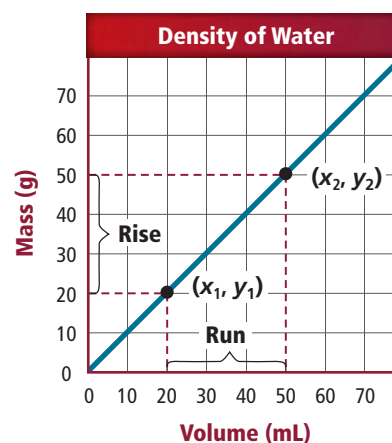$y \propto \sqrt[n]{x}$
$(n > 1)$

■ **Figure 13** The shape of the curve formed by a plot of experimental data indicates how the variables are related.

■ **Figure 14** A steep slope indicates that the dependent variable changes rapidly with a change in the independent variable.

Infer *What would an almost flat line indicate?*

The linear graph

The linear graph is useful in analyzing data because a linear relationship can be translated easily into equation form using the equation for a straight line.

$$y = mx + b$$

In the equation, y stands for the dependent variable, m is the slope of the line, x stands for the independent variable, and b is the y-intercept, the point where the curve crosses the y-axis.

The slope of a linear graph is the steepness of the line. Slope is defined as the ratio of the vertical change (the rise) to the horizontal change (the run) as you move from one point to the next along the line. Use the graph in **Figure 14** to calculate slope. Choose any two points on the line, (x_1, y_1) and (x_2, y_2). The two points need not be actual data points, but both must fall somewhere on the straight line. After selecting two points, calculate slope, m, using the following equation.

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2$$

The symbol Δ stands for change, x_1 and y_1 are the coordinates or values of the first point, and x_2 and y_2 are the coordinates of the second point.
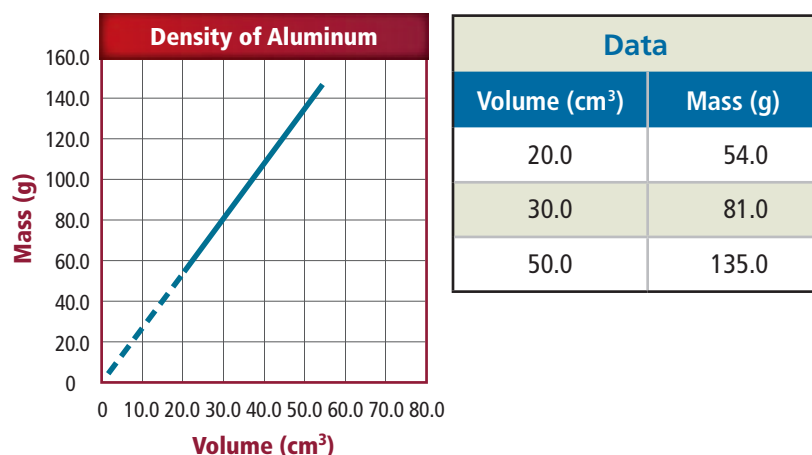
Choose any two points along the graph of mass v. volume in **Figure 15,** and calculate its slope.

$$m = \frac{135 \text{ g} - 54 \text{ g}}{50.0 \text{ cm}^3 - 20.0 \text{ cm}^3} = 2.7 \text{ g/cm}^3$$

Note that the units for the slope are the units for density. Plotting a graph of mass versus volume is one way of determining the density of a substance.

Apply the general equation for a straight line to the graph in **Figure 15.**

$$y = mx + b$$
$$mass = (2.7 \text{ g/cm}^3)(volume) + 0$$
$$mass = (2.7 \text{ g/cm}^3)(volume)$$

Density of Aluminum		Data	
		Volume (cm³)	**Mass (g)**
		20.0	54.0
		30.0	81.0
		50.0	135.0

Once the data from the graph in **Figure 15** has been placed in the general equation for a straight line, this equation verifies the direct relationship between mass and volume. For any increase in volume, the mass also increases.

Interpolation and extrapolation

Graphs also serve functions other than determining the relationship between variables. They permit interpolation, the prediction of values of the independent and dependent variables. For example, you can see in the table in **Figure 15** that the mass of 40.0 cm³ of aluminum was not measured. However, you can interpolate from the graph that the mass would be 108 g.

Graphs also permit extrapolation, which is the determination of points beyond the measured points. To extrapolate, draw a broken line to extend the curve to the desired point. In **Figure 15,** you can determine that the mass at 10.0 cm³ equals 27 g. One caution regarding extrapolation—some straight-line curves do not remain straight indefinitely. So, extrapolation should only be done where there is a reasonable likelihood that the curve does not change.

PRACTICE Problems

17. Plot the data in each table. Explain whether the graphs represent direct or inverse relationships.

Table 3 Gas Pressure and Volume		Table 4 Gas Pressure and Temperature	
Pressure (mm Hg)	**Volume (mL)**	**Pressure (mm Hg)**	**Temperature (K)**
3040	5.0	3040	1092
1520	10.0	1520	546
1013	15.0	1013	410
760	20.0	760	273

■ **Figure 16** The mass of one lime would be one-twelfth the mass of one dozen limes.

Ratios, Fractions, and Percents

When you analyze data, you may be asked to compare measured quantities. Or, you may be asked to determine the relative amounts of elements in a compound. Suppose, for example, you are asked to compare the molar masses of the diatomic gases, hydrogen (H_2) and oxygen (O_2). The molar mass of hydrogen gas equals 2.00 g/mol; the molar mass of oxygen equals 32.00 g/mol. The relationship between molar masses can be expressed in three ways: a ratio, a fraction, or a percent.

Ratios

You make comparisons by using ratios in your daily life. For example, if the mass of a dozen limes is shown in **Figure 16,** how does it compare to the mass of one lime? The mass of one dozen limes is 12 times larger than the mass of one lime. In chemistry, the chemical formula for a compound compares the elements that make up that compound, as shown in **Figure 17.** A ratio is a comparison of two numbers by division. One way it can be expressed is with a colon (:). The comparison between the molar masses of oxygen and hydrogen can be expressed as follows.

$$\text{molar mass (H}_2\text{):molar mass (O}_2\text{)}$$
$$2.00 \text{ g/mol:}32.00 \text{ g/mol}$$
$$2.00\text{:}32.00$$
$$1\text{:}16$$

Notice that the ratio 1:16 is the smallest integer (whole number) ratio. It is obtained by dividing both numbers in the ratio by the smaller number, and then rounding the larger number to remove the digits after the decimal. The ratio of the molar masses is 1 to 16. In other words, the ratio indicates that the molar mass of diatomic hydrogen gas is 16 times smaller than the molar mass of diatomic oxygen gas.

■ **Figure 17** In a crystal of table salt (sodium chloride), each sodium ion is surrounded by chloride ions, yet the ratio of sodium ions to chloride ions is 1:1. The formula for sodium chloride is NaCl.

Fractions

Ratios are often expressed as fractions in simplest form. A fraction is a quotient of two numbers. To express the comparison of the molar masses as a fraction, place the molar mass of hydrogen over the molar mass of oxygen as follows.

$$\frac{\text{molar mass H}_2}{\text{molar mass O}_2}$$
$$= \frac{2.0 \text{ g/mol}}{32.00 \text{ g/mol}}$$
$$= \frac{2.00}{32.00}$$
$$= \frac{1}{16}$$

In this case, the simplified fraction is calculated by dividing both the numerator (top of the fraction) and the denominator (bottom of the fraction) by 2.00. This fraction yields the same information as the ratio. That is, diatomic hydrogen gas has one-sixteenth the mass of diatomic oxygen gas.

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Percents

A percent is a ratio that compares a number to 100. The symbol for percent is %. You also are used to working with percents in your daily life. The number of correct answers on an exam can be expressed as a percent. If you answered 90 out of 100 questions correctly, you would receive a grade of 90%. Signs like the one in **Figure 18** indicate a reduction in price. If the item's regular price is $100, how many dollars would you save? Sixty percent means 60 of every 100, so you would save $60. How much would you save if the sign said 75% off?

The comparison between molar mass of hydrogen gas and the molar mass of oxygen gas described on the previous page can also be expressed as a percent by taking the fraction, converting it to decimal form, and multiplying by 100 as follows.

$$\frac{\text{molar mass H}_2}{\text{molar mass O}_2} \times 100 = \frac{2.00 \text{ g/mol}}{32.00 \text{ g/mol}} \times 100 = 0.0625 \times 100 = 6.25\%$$

Diatomic hydrogen gas has 6.25% of the mass of diatomic oxygen gas.

Operations Involving Fractions

Fractions are subject to the same type of operations as other numbers. Remember that the number on the top of a fraction is the numerator and the number on the bottom is the denominator. **Figure 19** shows an example of a fraction.

1. Addition and subtraction

Before two fractions can be added or subtracted, they must have a common denominator. Common denominators are found by finding the least common multiple of the two denominators. Finding the least common multiple is often as easy as multiplying the two denominators together. For example, the least common multiple of the denominators of the fractions $\frac{1}{2}$ and $\frac{1}{3}$ is 2×3 or 6.

$$\frac{1}{2} + \frac{1}{3} = \left(\frac{3}{3} \times \frac{1}{2}\right) + \left(\frac{2}{2} \times \frac{1}{3}\right) = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Sometimes, one of the denominators will divide into the other, which makes the larger of the two denominators the least common multiple. For example, the fractions $\frac{1}{2}$ and $\frac{1}{6}$ have 6 as the least common multiple denominator.

$$\frac{1}{2} + \frac{1}{6} = \left(\frac{3}{3} \times \frac{1}{2}\right) + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6}$$

In other situations, both denominators will divide into a number that is not the product of the two. For example, the fractions $\frac{1}{4}$ and $\frac{1}{6}$ have the number 12 as their least common multiple denominator, rather than 24, the product of the two denominators.

The least common denominator can be deduced as follows:

$$\frac{1}{6} + \frac{1}{4} = \left(\frac{4}{4} \times \frac{1}{6}\right) + \left(\frac{6}{6} \times \frac{1}{4}\right) = \frac{4}{24} + \frac{6}{24} = \frac{2}{12} + \frac{3}{12} = \frac{5}{12}$$

Because both fractions can be simplified by dividing numerator and denominator by 2, the least common multiple must be 12.

■ **Figure 18** Stores often use percentages when advertising sales.
Analyze *Would the savings be large at this sale? How would you determine the sale price?*

■ **Figure 19** When two numbers are divided, the one on top is the numerator and the one on the bottom is the denominator. The result is called the quotient. When you perform calculations with fractions, the quotient can be expressed as a fraction or a decimal.

Dividend (numerator)

$$\text{Quotient} = \frac{9 \times 10^8}{3 \times 10^{-4}}$$

Divisor (denominator)

2. Multiplication and division

When multiplying fractions, the numerators and denominators are multiplied together as follows:

$$\frac{1}{2} \times \frac{2}{3} = \frac{1 \times 2}{2 \times 3} = \frac{2}{6} = \frac{1}{3}$$

Note the final answer is simplified by dividing the numerator and denominator by 2.

When dividing fractions, the divisor is inverted and multiplied by the dividend as follows:

$$\frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \times \frac{2}{1} = \frac{2 \times 2}{3 \times 1} = \frac{4}{3}$$

PRACTICE Problems

18. Perform the indicated operation:

a. $\frac{2}{3} + \frac{3}{4}$

b. $\frac{4}{5} + \frac{3}{10}$

c. $\frac{1}{4} - \frac{1}{6}$

d. $\frac{7}{8} - \frac{5}{6}$

e. $\frac{1}{3} \times \frac{3}{4}$

f. $\frac{3}{5} \times \frac{2}{7}$

g. $\frac{5}{8} \div \frac{1}{4}$

h. $\frac{4}{9} \div \frac{3}{8}$

Logarithms and Antilogarithms

When you perform calculations, such as the pH of the products in **Figure 20,** you might need to use the log or antilog function on your calculator. A logarithm (log) is the power or exponent to which a number, called a base, must be raised in order to obtain a given positive number.

This textbook uses common logarithms based on a base of 10. Therefore, the common log of any number is the power to which 10 is raised to equal that number. Examine **Table 5** to compare logs and exponents. Note the log of each number is the power of 10 for the exponent of that number. For example, the common log of 100 is 2, and the common log of 0.01 is -2.

$$\log 10^2 = 2$$
$$\log 10^{-2} = -2$$

A common log can be written in the following general form.

If $10^n = y$, then $\log y = n$.

In each example in **Table 5,** the log can be determined by inspection. How do you express the common log of 5.34×10^5? Because logarithms are exponents, they have the same properties as exponents, as shown in **Table 6** on the next page.

$$\log 5.34 \times 10^5 = \log 5.34 + \log 10^5$$

Table 5 Comparison Between Exponents and Logs

Exponent	Logarithm
$10^0 = 1$	$\log 1 = 0$
$10^1 = 10$	$\log 10 = 1$
$10^2 = 100$	$\log 100 = 2$
$10^{-1} = 0.1$	$\log 0.1 = -1$
$10^{-2} = 0.01$	$\log 0.01 = -2$

Table 6 Properties of Exponents	
Exponential Notation	**Logarithm**
$10^A \times 10^B = 10^{A+B}$	$\log(A \times B) = \log A + \log B$
$10^A \div 10^B = 10^{A-B}$	$\log(A \div B) = \log A - \log B$
A^B	$(\log A) \times B$

Significant figures and logarithms

Most scientific calculators have a button labeled *log* and, in most cases, you enter the number and push the log button to display the log of the number. Note that there is the same number of digits after the decimal in the log as there are significant figures in the original number entered.

$$\log 5.34 \times 10^5 = \log 5.34 + \log 10^5 = 0.728 + 5 = 5.728$$

Antilogarithms

Suppose the pH of the aqueous ammonia in **Figure 20** is 9.54 and you are asked to find the concentration of the hydrogen ions in that solution. By definition, $pH = -\log[H^+]$. Compare this to the general equation for the common log.

$$\text{Equation for pH:} \qquad pH = -\log[H^+]$$
$$\text{General equation:} \qquad y = \log 10^n$$

To solve the equation for $[H^+]$, you must follow the reverse process and calculate the antilogarithm (antilog) of -9.54 to find $[H^+]$.

Antilogs are the reverse of logs. To find the antilog, use a scientific calculator to input the value of the log. Then, use the inverse function and press the log button. The number of digits after the decimal in the log equals the number of significant figures in the antilog. An antilog can be written in the following general form.

$$\text{If } n = \text{antilog } y, \text{ then } y = 10^n.$$
$$\text{Thus, } [H^+] = \text{antilog}(-9.54) = 10^{-9.54} = 10^{(0.46-10)}$$
$$= 10^{0.46} \times 10^{-10}$$
$$= 2.9 \times 10^{-10}M$$

Check the instruction manual for your calculator. The exact procedure to calculate logs and antilogs might vary.

■ **Figure 20** Ammonia is a base. That means its hydrogen ion concentration is less than $10^{-7}M$.

PRACTICE Problems

19. Find the log of each of the following numbers.

    a. 367   **b.** 4078   **c.** X^n

20. Find the antilog of each of the following logs.

    a. 4.663   **b.** 2.367   **c.** 0.371   **d.** -1.588